LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2024**

PST1MC01 – ADVANCED DISTRIBUTION THEORY

Date: 08-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Define the following**
a)	Hypergeometric distribution.
b)	Trinomial distribution.
c)	Non-central F distribution.
d)	Order Statistics.
e)	Quadratic form.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**Fill in the blanks**
a)	The variance of beta distribution of second kind is_____________.
b)	The characteristic function of bivariate Poisson distribution is____________.
c)	The mean of non-central t-distribution is_______________.
d)	The asymptotic distribution of the kth order statistic from a normal distribution is _______________.
e)	The trace of the matrix A equals the sum of the________________.
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Establish the recurrence relation for the moments of binomial distribution and obtain its variance.
4	Derive the probability mass function and conditional distribution of the bivariate poisson distribution.
5	If $X \sim \beta_1(\mu, v)$ and $Y \sim \gamma(\lambda, \mu+v)$ are independent random variables$(\mu, v, \lambda > 0)$, find a p.d.f for XY and identify its distribution.
6	If a random sample of size n (=2k+1 an odd number) from $U(0, \theta)$, find the mean, variance and the median of the distribution.
7	(i) If X_1, X_2, X_3, X_4 are iid $N(0, \sigma^2)$ random variables, obtain the distribution of $X_1X_2 - X_3X_4$. (ii)Using quadratic forms, show that if $X_1, X_2, \ldots, X_n$ are iid $N(0, \sigma^2)$ random variables $\sum \frac{(X_i - \bar{X})^2}{\sigma^2} \sim \aleph^2(n-1)$. **(5+5)**
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	(i)State and prove memoryless property of geometric distribution. (ii)If two normal universes A and B have the same total frequency but the standard deviation of universe A is k times that of the universe of B, show that maximum frequency of universe A is (1/k) times that of universe B. **(9+3.5)**
9	Establish the joint p.d.f of kth order statistics.
10	Derive the mean and variance of non-central ‘t’ distribution.
11	If X and Y are i.i.d standard Cauchy variates, prove that the p.d.f of U = XY is $\frac{2}{\pi^2}\left\{\frac{\log\lvert u\rvert}{u^2-1}\right\}$; $-\infty < u < \infty$.

	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	If $X_1, X_2, \dots, X_n$ are iid $N(0, \sigma^2), \sigma > 0$, define $X = (X_1, X_2, \dots, X_n)'$ and $Q = X'AX$ with $\rho(A) = r$. Then $\frac{Q}{\sigma^2}$ is distributed as chi-square distribution with r degrees of freedom iff A is idempotent.
13	Derive the marginal, conditional distributions and regression coefficients for bivariate normal distribution.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	Derive the p.d.f of non-central chi-square distribution.
15	(i)Derive the **rth** moments of beta distribution of first and second kind and determine mean and variance of the distribution. (ii) Prove that mean = median = mode = μ for normal distribution **(12+8)**

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