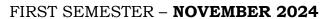
## LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034



## M.Sc. DEGREE EXAMINATION – STATISTICS





## PST1MC01 - ADVANCED DISTRIBUTION THEORY

| _  | Date: 08-11-2024 Dept. No.  | Max.: 100 Marks                   |
|----|---|-----------------------------------|
| T  | Time: 01:00 pm-04:00 pm   |                                   |
|    |   |                                   |
|    | SECTION A – K1 (CO1)  |                                   |
|    | Answer ALL the questions  | $(5 \times 1 = 5)$                |
| 1  | Define the following  |                                   |
| a) | Hypergeometric distribution.  |                                   |
| b) | Trinomial distribution.   |                                   |
| c) | Non-central F distribution.   |                                   |
| d) | Order Statistics.   |                                   |
| e) | Quadratic form.   |                                   |
|    | SECTION A – K2 (CO1)  |                                   |
|    | Answer ALL the questions  | $(5 \times 1 = 5)$                |
| 2  | Fill in the blanks  | $(S \mathbf{A} \mathbf{I} - S)$   |
| a) | The variance of beta distribution of second kind is   |                                   |
| b) | The characteristic function of bivariate Poisson distribution is  |                                   |
| c) | The mean of non-central t-distribution is   | <del></del>                       |
| d) | The asymptotic distribution of the k <sup>th</sup> order statistic from a normal distribution is  |                                   |
| e) | The trace of the matrix A equals the sum of the   |                                   |
|    | SECTION B – K3 (CO2)  |                                   |
|    |   |                                   |
|    | Answer any THREE of the following   | $(3 \times 10 = 30)$              |
| 3  | Establish the recurrence relation for the moments of binomial distribution and obtain its variance.   |                                   |
| 4  | Derive the probability mass function and conditional distribution of the bivariate poisson distribution.  |                                   |
| 5  | If $X \sim \beta_1(\mu, \nu)$ and $Y \sim \gamma(\lambda, \mu + \nu)$ are independent random variables $(\mu, \nu, \lambda > 0)$ , find a p.d.f for XY and identify its distribution. |                                   |
| 6  | If a random sample of size n (=2k+1 an odd number) from $U(0, \theta)$ , find the mean, variance and the median of the distribution.  |                                   |
| 7  | (i) If $X_1, X_2, X_3, X_4$ are iid $N(0, \sigma^2)$ random variables, obtain the distribution of $X_1X_2 - X_3X_4$ .   |                                   |
|    | (ii) Using quadratic forms, show that if $X_1, X_2,, X_n$ are iid   | $N(0, \sigma^2)$ random variables |
|    | $\sum \frac{(X_i - \bar{X})^2}{\sigma^2} \sim \aleph^2(n-1).$   | (5+5)                             |
|    | SECTION C – K4 (CO3)  |                                   |
|    | Answer any TWO of the following   | $(2 \times 12.5 = 25)$            |
| 8  | (i)State and prove memoryless property of geometric distribution.   |                                   |
|    | (ii)If two normal universes A and B have the same total frequency but th  |                                   |
|    | A is k times that of the universe of B, show that maximum frequency of  |                                   |
|    | universe B.   | (9+3.5)                           |
| 9  | Establish the joint p.d.f of k <sup>th</sup> order statistics.  |                                   |
| 10 | Derive the mean and variance of non-central 't' distribution.   |                                   |
| 11 | If X and Y are i.i.d standard Cauchy variates, prove that the p.d.f of $U = XY$ is  |                                   |
|    | $\left  \frac{2}{\pi^2} \left\{ \frac{\log  u }{u^2 - 1} \right\}; -\infty < u < \infty.$   |                                   |

| SECTION D – K5 (CO4) |   |  |
|----------------------|---|--|
|                      | Answer any ONE of the following $(1 \times 15 = 15)$  |  |
| 12                   | If $X_1, X_2, \dots, X_n$ are iid $N(0, \sigma^2), \sigma > 0$ , define $X = (X_1, X_2, \dots, X_n)'$ and $Q = X'AX$ with $\rho(A) = r$ . |  |
|                      | Then $\frac{Q}{\sigma^2}$ is distributed as chi-square distribution with r degrees of freedom iff A is idempotent.                        |  |
| 13                   | Derive the marginal, conditional distributions and regression coefficients for bivariate normal   |  |
|                      | distribution.   |  |
| SECTION E – K6 (CO5) |   |  |
|                      | Answer any ONE of the following $(1 \times 20 = 20)$  |  |
| 14                   | Derive the p.d.f of non-central chi-square distribution.  |  |
| 15                   | (i)Derive the <b>r</b> <sup>th</sup> moments of beta distribution of first and second kind and determine mean and variance                |  |
|                      | of the distribution.  |  |
|                      | (ii) Prove that mean = median = mode = $\mu$ for normal distribution (12+8)   |  |

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